MATH 119: Midterm 1

Name: ______________________________

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
		60

1. Short answer questions:

   (a) Suppose you write

   $$(x+y)^2z^2 = x^2 + y^2z^2$$

   What are the two errors you made?

   ① x and y are terms. can only manipulate exponents (exponent laws) across factors.

   ② everything to the left of z^2 should be encapsulated in parentheses since you are multiplying z^2 into ≥ 2 terms

   (b) True or false: We can simplify $\frac{x^2+x-2}{x-1}$ by crossing out the x's to become $\frac{x^2-2}{-1}$. If not, properly simplify the expression.

   False; x is both a term in the context of the entire numerator and dinominator.

   $$\frac{x^2+x-2}{x-1} = \frac{\cancel{(x-1)}\cdot(x+2)}{\cancel{(x-1)}} = \boxed{x+2}$$

   (c) Bob has a function $f(x)$. It is not one-to-one. However, he goes ahead and finds the inverse f^{-1}. **What** is the problem with f^{-1} and **why**?

   f^{-1} is not a function because one input is sent to at least two diffferent outputs.

   (d) If $f(x) = \frac{x}{1-x}$, find $f(x^2-1)$.

   $$f(x^2-1) = \frac{x^2-1}{1-(x^2-1)} = \frac{x^2-1}{1-x^2+1} = \frac{x^2-1}{2-x^2}$$

   (e) Suppose we have a base function $f(x) = x^3$ and we have

   $$g(x) = (x+2)^3 + 4 \qquad h(x) = \left(\frac{1}{2}x+2\right)^3 + 4 = \left(\frac{1}{2}(x+4)\right)^3 + 4$$

   Does $g(x)$ have the same horizontal shift as $h(x)$? If not, state what **both** $g(x)$ and $h(x)$'s horizontal shift are.

No. g(x) is 2 units to the left of f(x) while h(x) is 4 units to the left of f(x).

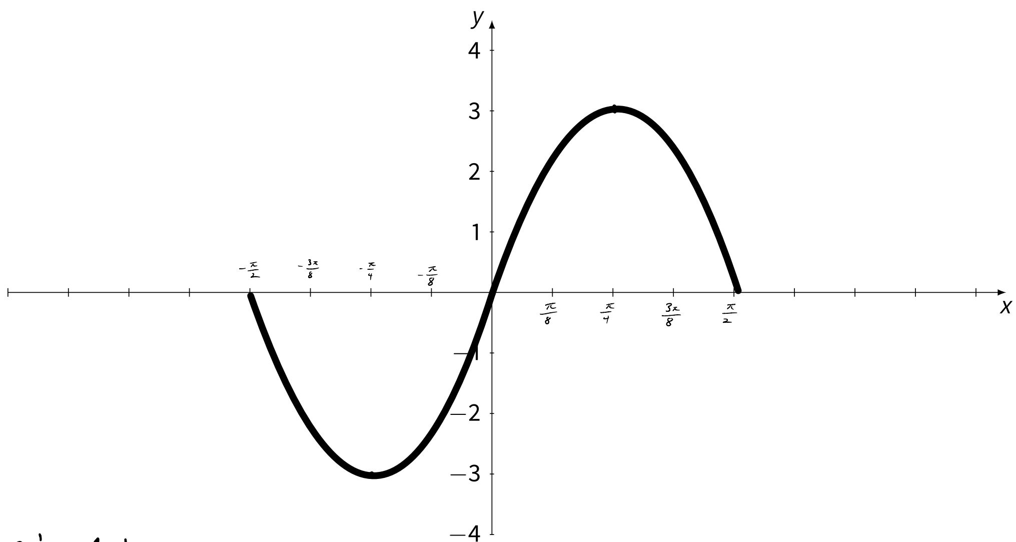
2. Suppose

$$f(x) = -3\sin(2x+\pi) = -3\sin\left(2\left(x+\frac{\pi}{2}\right)\right)$$

Do two things:

(a) Graph at least one period of $f(x)$ using transformations. Label the x-axis tick marks you are using.

(b) Write out the blueprint of transformations starting with $g(x) = \sin x$ to end up at $f(x)$.



$g(x) = \sin(x)$

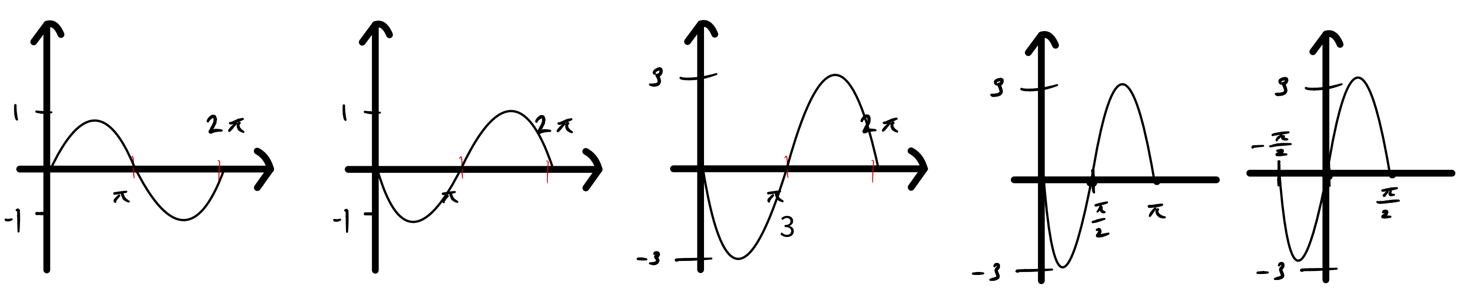
$h(x) = -g(x) \quad = -\sin(x)$ reflection around x-axis

$j(x) = 3h(x) \quad = -3\sin(x)$ v. stretch 3 units

$k(x) = j(2x) \quad = -3\sin(2x)$ h. shrink factor of 1/2

$f(x) = k\left(x+\frac{\pi}{2}\right) = -3\sin\left(2\left(x+\frac{\pi}{2}\right)\right)$ h. shift $\frac{\pi}{2}$ left

$g(x) \longrightarrow h(x) \longrightarrow j(x) \longrightarrow k(x) \longrightarrow f(x)$



$2 \quad -1$

$1 \quad -3$

3. Let

$$f(x) = 2x^2 - 7x + 3 \qquad g(x) = \sin(x) - \frac{1}{x-1}$$

(a) Factor $f(x)$.

$$\boxed{(2x-1)(x-3)}$$

(b) Find and simplify $f(x) - g(x)$ and it's domain given in interval notation.

$$f(x) - g(x) = 2x^2 - 7x + 3 - \left(\sin(x) - \frac{1}{x-1}\right)$$

$$= \boxed{2x^2 - 7x + 1 - \sin(x) + \frac{1}{x-1}, \quad (-\infty, 1) \cup (1, \infty)}$$

problem at $x = 1$

(c) Evaluate and simplify $f(x+h) - f(x)$ (you should be able to factor out h at the end).

$$f(x+h) - f(x) = 2(x+h)^2 - 7(x+h) + 3 - (2x^2 - 7x + 3)$$

$$= 2(x^2 + 2xh + h^2) \cancel{-7x} - 7h \cancel{+3} - 2x^2 \cancel{+7x} \cancel{-3}$$

$$= \cancel{2x^2} + 4xh + 2h^2 - 7h \cancel{-2x^2}$$

$$= \boxed{h \cdot (4x + 2h - 7)}$$

4. Given $ax - bx(c+d) - ex = gx$, isolate x.

$ax - bcx - bdx - ex = gx$

$ax - bcx - bdx - ex - gx = 0$

$$\frac{x\cdot(a - bc - bd - e - g)}{a - bc - bd - e - g} = \frac{0}{a - bc - bd - e - g}$$

$$\boxed{x = 0}$$

5. Solve for x:

$$\frac{10}{x} - \frac{12}{x-3} + 4 = 0$$

LCD : $x(x-3)$

$$x(x-3)\left(\frac{10}{x} - \frac{12}{x-3} + 4\right) = 0\cdot x(x-3)$$

$$\frac{10\cancel{x}(x-3)}{\cancel{x}} - \frac{12x\cancel{(x-3)}}{\cancel{(x-3)}} + 4x(x-3) = 0$$

$$10x - 30 - 12x + 4x^2 - 12x = 0$$

$$4x^2 - 12x - 30 = 0$$

2 3
2 −10

$$(2x+3)(2x-10) = 0$$

cont.

$$2x + 3 = 0 \qquad\qquad 2x - 10 = 0$$

$$x = -\frac{3}{2} \qquad\qquad 2x = 10$$

$$x = -\frac{3}{2} \qquad\qquad x = 5$$

Check $x = -\frac{3}{2}$

$$\frac{10}{-\frac{3}{2}} - \frac{12}{-\frac{3}{2} - 3} + 4 = 10 \cdot \left(-\frac{2}{3}\right) - \frac{12}{-\frac{3}{2} - \frac{6}{2}} + 4$$

$$= -\frac{20}{3} - \frac{12}{-\frac{9}{2}} + 4$$

$$= -\frac{20}{3} + \cancel{12}^{4} \cdot \frac{2}{\cancel{9}_{3}} + 4$$

$$= -\frac{20}{3} + \frac{8}{3} + \frac{12}{3}$$

$$= \frac{-20 + 20}{3}$$

$$= 0 \checkmark$$

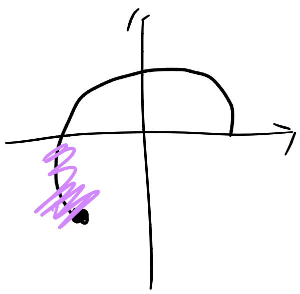
$$\boxed{\text{solution: } x = -\frac{3}{2} \quad x = 5}$$

Check $x = 5$

$$\frac{10}{5} - \frac{12}{5-3} + 4 = 2 - \frac{12}{2} + 4 = 2 - 6 + 4 = 0 \checkmark$$

6. Evaluate the following trigonometric functions:

(a) $\sin\left(\frac{5\pi}{4}\right)$

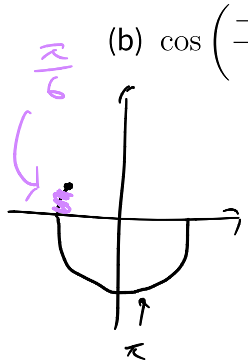


① $\bar{t} = \frac{\pi}{4}$

② sin negative in III

$$\sin\left(\frac{5\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$$

(b) $\cos\left(\frac{-7\pi}{6}\right)$



① $\bar{t} = \frac{\pi}{6}$

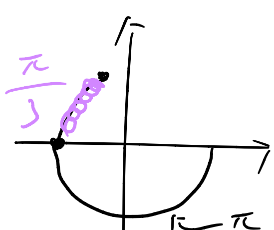
② cos negative in II

$$\cos\left(-\frac{7\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = \boxed{-\frac{\sqrt{3}}{2}}$$

(c) $\tan\left(\frac{-40\pi}{3}\right) = \tan\left(-\frac{39\pi}{3} - \frac{\pi}{3}\right)$

$= \tan\left(-13\pi - \frac{\pi}{3}\right)$

$2\cdot(-6\pi) - \pi$ ignore



① $\bar{t} = \frac{\pi}{3}$

② tan negative in II

$$\tan\left(-\frac{40\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right)$$

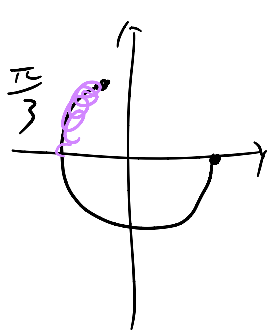
$$= -\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2}\cdot\frac{2}{1}$$

$$= \boxed{-\sqrt{3}}$$

(d) $\csc\left(10000000000000\pi - \frac{4\pi}{3}\right)$

ignore

$= \csc\left(-\frac{4\pi}{3}\right)$



① $\bar{t} = \frac{\pi}{3}$

② csc positive in II since sin positive in II

$$\csc\left(-\frac{4\pi}{3}\right) = \csc\left(\frac{\pi}{3}\right) = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

$$= \boxed{\frac{2\sqrt{3}}{3}}$$